

(Please write your Exam Roll No.)

Exam Roll No.

END TERM EXAMINATION

FIRST SEMESTER [BCA] NOVEMBER-DECEMBER 2017

Paper Code: BCA-101

Subject: Mathematics-I

Time: 3 Hours

Maximum Marks: 75

Note: Attempt any five questions including Q.no.1 which is compulsory.
Select one question from each unit.

- Q1 (a) Solve the following system of equations by Cramer's rule:
 $2y - 3z = 0, \quad x + 3y = -4, \quad 3x + 4y = 3.$ (5)
- (b) Solve: $\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0.$ (5)
- (c) Find the maximum and minimum values of $f(x) = x + \sin 2x$ in $[0, 2\pi]$. (5)
- (d) Evaluate $\int \frac{\sqrt{\tan x}}{\sin x \cos x} dx.$ (5)
- (e) Show that $\lim_{x \rightarrow 0} \frac{e^{\frac{1}{x}} - 1}{\frac{1}{e^x} + 1}$ does not exist. (5)

Unit-I

- Q2 (a) Find eigen values and eigen vectors of $A = \begin{bmatrix} 2 & 0 & 4 \\ 0 & 6 & 0 \\ 4 & 0 & 2 \end{bmatrix}.$ (6.5)
- (b) Find whether or not the following set of vectors are linearly dependent or independent.
 $X_1 = (1, 1, 0), \quad X_2 = (1, 0, 1), \quad X_3 = (0, 1, 1).$ (6)
- Q3 (a) Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and hence find $A^{-1}.$ (6.5)
- (b) Find the rank of matrix $\begin{bmatrix} 1 & 1 & 2 & 3 \\ 3 & 4 & 7 & 10 \\ 5 & 7 & 11 & 17 \\ 6 & 8 & 13 & 16 \end{bmatrix}.$ (6)

Unit-II

- Q4 (a) Discuss the continuity of the function $f(x) = \begin{cases} -x, & x \leq 0 \\ x, & 0 < x \leq 1 \\ 2-x, & 1 < x \leq 2 \\ 1, & x > 2 \end{cases}$ at each point $x = 0, 1, 2.$ (6.5)
- (b) $\lim_{x \rightarrow 0} \left(\frac{\cos mx - \cos nx}{x^2} \right).$ (6)

- Q5 (a) Let $f(x) = \begin{cases} 1, & x \leq 3 \\ ax + b, & 3 < x < 5 \\ 7, & 5 \leq x \end{cases}$. Find the values of a and b so that $f(x)$ is continuous. (6.5)
- (b) Evaluate: (i) $\lim_{x \rightarrow \infty} \frac{x}{\sqrt{4x^2 + 1}}$ (ii) $\lim_{x \rightarrow 0} \frac{x^3 \cot x}{1 - \cos x}$. (6)

Unit-III

- Q6 (a) Verify the hypothesis and conclusion of Lagrange's mean value theorem for the function $f(x) = \frac{1}{4x-1}$, $1 \leq x \leq 4$. (6.5)
- (b) Expand $\log \sin x$ in powers of $(x-2)$ by Taylor's series. (6)
- Q7 (a) If $y = [x + \sqrt{1+x^2}]^m$, show that $(1+x^2)y_{n+2} + (2n+1)xy_{n+1} + (n^2 - m^2)y_n = 0$. Also find $y_n(0)$. (6.5)
- (b) Find asymptotes of the curve $x^3 + 2x^2y - xy^2 - 2y^3 + x^2 - y^2 - 2x - 3y = 0$. (6)

Unit-IV

- Q8 (a) Evaluate: (i) $\int \frac{x \tan^{-1} x}{(1+x^2)^{3/2}} dx$ (ii) $\int e^x \left(\frac{1-\sin x}{1-\cos x} \right) dx$. (6)
- (b) Show that $\beta(p, q) = \int_0^1 \frac{x^{p-1} + x^{q-1}}{(1+x)^{p+q}} dx$. (6.5)
- Q9 (a) If $I_{m,n} = \int \cos^m x \sin nx dx$, prove that $(m+n)I_{m,n} = -\cos^m x \cos nx + mI_{m-1,n-1}$.
Hence evaluate $\int_0^{\pi/2} \cos^5 x \sin 3x dx$. (7.5)
- (b) Evaluate $\int_0^1 x^{3/2} (1-x)^{3/2} dx$. (5)
