

END TERM EXAMINATION

FIRST SEMESTER [BCA] DEC.2014 - JAN.2015

Paper Code: BCA101

Subject: Mathematics-I
(Batch: 2011 onwards)

Time : 3 Hours

Maximum Marks :75

Note: Attempt any five questions including Q.no.1 which is compulsory.
Select one question from each unit.

Q1 (a) If $A = \begin{bmatrix} 9 & 1 \\ 4 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 5 \\ 7 & 12 \end{bmatrix}$ find the matrix X such that $3A+5B+2X=0$. (3)

(b) Prove that if (verify by finding AA^{-1}) $A = \begin{bmatrix} 2 & 3 \\ 4 & 2 \end{bmatrix}$ then $A^{-1} = \begin{bmatrix} -1/4 & 3/8 \\ 1/2 & 1/4 \end{bmatrix}$ (3)

(c) Show that $y = \frac{x^2-1}{x-1}$ is continuous except at $x=1$. What is the nature of the discontinuity? (3)

(d) Find $\lim_{x \rightarrow 0} \frac{\ln \sqrt{x+1}-1}{x}$. $= \frac{1}{2}$ (3)

(e) Using Taylor's series, find the value of $f\left(\frac{21}{20}\right)$ if $f(x) = x^3 - 6x^2 + 7$. (3)

(f) Show that $\sin(x)(1+\cos x)$ is maximum when $x = \frac{\pi}{3}$. $\sqrt{5/2}$ (3)

(g) Evaluate $\int e^x \left(\frac{x-1}{x^2}\right) dx$. (3)

(h) Evaluate $\int e^x \cos^2 x dx$. (4)

UNIT-I

Q2 (a) Given $A = \begin{bmatrix} 2 & 1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$, find A^{-1} and A^4 using Cayley-Hamilton

Theorem. (6)

(b) If $A = \begin{bmatrix} 1 & 3 & 0 \\ -1 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 2 & 3 \\ -1 & 1 & 2 \end{bmatrix}$ compute AB and BA and show that

$AB \neq BA$. (6.5)

Q3 (a) If the matrix $\begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix}$ is orthogonal, then find the values of a, b

and c. (6)

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- (b) Determine the rank of the following matrix using elementary row transformation:- (6.5)

$$\begin{bmatrix} 2 & 1 & -3 & 4 \\ 2 & 4 & -2 & 5 \\ 0 & 3 & 1 & 3 \\ 2 & 1 & -3 & -2 \end{bmatrix}$$

UNIT-II

- Q4 (a) For what value of x does $y = \frac{x+1}{(x+2)(x+3)}$ tends to infinity? Indicate the form of the graph of the function and describe its discontinuities. (6)

- (b) Evaluate $\lim_{m \rightarrow \infty} P \left(1 + \frac{i}{m} \right)^{mn}$. (6.5)

- Q5 (a) A function f is defined as follows:- $f(x) = \begin{cases} \frac{9x}{x+2}, & \text{if } x < 1 \\ 3, & \text{if } x = 1 \\ \frac{x+3}{x}, & \text{if } x > 1 \end{cases}$. Examine the continuity of f in the interval $(-3, 3)$. (6)

- (b) Find the value of a so that the function $f(x) = \begin{cases} ax+5 & \text{if } x \leq 2 \\ x-1 & \text{if } x > 2 \end{cases}$ is continuous at $x=2$. (6.5)

UNIT-III

- Q6 (a) Find $\frac{dy}{dx}$ if-
(i) $y = \sin \sqrt{x}$ (ii) $x^y \cdot y^x = K$ where K is a constant. (iii) $y = \sin^3 2x$. (6)
(b) Find all the asymptotes of the curve $y^2(x-2a) = x^3 - a^2$. (6.5)

- Q7 (a) Find the n th derivative of $\log(2x+3)$. (6)
(b) Show that the function $f(x) = x^2 + \frac{250}{x}$ has a minimum value at $x=5$. (6.5)

UNIT-IV

- Q8 (a) Find the following integrals:- (6)
(i) $\int x e^{-x} dx$ (ii) $\int x^n \log x dx$.
(b) Find out the Reduction Formulae for $\int_0^{\pi/4} \sin^n x dx$, n being a positive integer. (6.5)

- Q9 (a) Prove that $\beta(m, n) = \beta(m+1, n) + \beta(m, n+1)$. (6)
(b) Evaluate $\int \frac{dx}{2x^2 + 3x + 5}$. (6.5)
